UNSTEADY-STATE SLIP OF A GAS NEAR AN INFINITE PLANE WITH DIFFUSION-MIRROR REFLECTION OF MOLECULES

M. M. Kuznetsov

UDC 533.7

On the basis of a "two-point" approximation for the distribution function, an approximate analytical solution is obtained to the Rayleigh problem describing (for an arbitrary moment of time) the unsteady-state slip of a rarefied gas near a surface with diffusion-mirror reflection of molecules. In the solution of the problem it was postulated that the characteristic value of the macroscopic velocity of the gas is small in comparison with the speed of sound. An approximate analogy is established with the propagation of free vibrations in an electrical line of infinite length. An investigation is made of the limiting transition $\sigma \to 0$ (σ is the fraction of mirror-reflected particles) in an exact solution of the Boltzmann problem. It is shown that, with $\sigma \ll 1$, the rate of slip for any given moment of time is determined from the hydrodynamic equations of motion. An investigation is made of the nonsingularity of the limiting transitions ($t \to \infty$, $\sigma \to 0$; $\sigma \to 0$, $t \to \infty$) with determination of the rate of thermal slip.

The Rayleigh Problem for a Moving Plane with Diffusion-Mirror Reflection of Molecules

Let us consider the problem of the unsteady-state motion of a gas, bounded by an infinite plane, which is suddenly set into uniform motion (parallel with itself) with a constant velocity w. A solution of this problem within the framework of the theory of a continuous medium has been given in [1, 2]. This unsteady-state flow was investigated in [3-9] on the basis of the kinetic theory of gases. The present article considers the case of the pulsed motion of an infinite plane with diffusion-mirror reflection of molecules. An approximate analytical solution is given to the problem which generalizes the solution obtained in [5] for the case of purely diffusion reflection of molecules.

A scheme of the diffusion-mirror reflection of molecules was also discussed in [3, 6, 9]. In distinction from the results of these communications, valid in the region of asymptotically small ($t/t_f \ll 1$) or large ($t/t_f \gg 1$) intervals of time t/t_f (t_f is the time of free flight), the proposed solution describes the behavior of the gas at an arbitrary moment of time* from the start of the motion of the infinite plane ($y=0$).

We assume that the velocity w of the plane is small in comparison with the speed of sound in the gas. In this case, the kinetic Boltzmann equation for the distribution function f(**c**, y, t) can be linearized with respect to the parameter w/v_W, ($v_W = \sqrt{2kT_W/m}$)

$$\partial\varphi/\partial t + c_y \partial\varphi/\partial y = \mathscr{L}[\varphi]. \tag{1}$$

Here φ is the correction to the equilibrium function $f^{(0)}$ for a gas at rest; $\varphi \sim w v_W^{-1}$; $f = f_W^{(0)}(1+\varphi)$; y is the distance along normal to the plane $y=0$; **c** is the velocity of a molecule; $\mathscr{L}[\varphi]$ is the linearized collision integral [10].

*In [3] an interpolation solution was constructed for the range $t/t_f \sim 1$. However, using this interpolation it was not possible to determine the velocity of the gas u(t, y) with $t/t_f \sim 1$ and $y>0$.

Moscow. Translated from Zhurnal Prikladnoi Mekhaniki i Tekhnicheskoi Fiziki, No. 6, pp. 19-25, November-December, 1975. Original article submitted October 21, 1974.

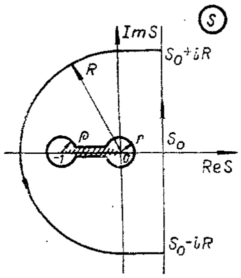

Fig. 1

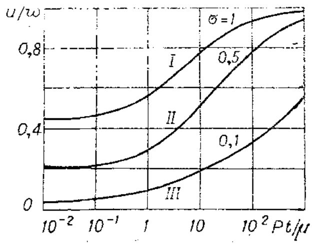

Fig. 2

As the kinetic boundary condition at the wall y = 0 we assume that part $(1-\sigma)$ of the incident molecules is reflected in a mirror fashion, while the other part (σ) is reflected diffusionally with a Maxwellian distribution at the temperature of the wall T_W. Then for the correction φ with y = 0 we will have

$$\varphi(c_y > 0,\ t,\ y=0)=(1-\sigma)\ \varphi(c_y < 0,\ t,\ y=0) + 2\sigma\bar{w}\zeta_x, \tag{2}$$

$$\text{where}\quad \bar{w} = uv_w^{-1};\quad \zeta_x = c_x v_w^{-1};$$

c_x is the projection of the velocity on a direction parallel to the plane y = 0.

For an approximate solution of Eq. (1), we use the following approximation of the function φ [5]:

$$\varphi = \begin{cases} \varphi^+ = \varphi\,(c_y > 0) = a_0^+\,(y)\,\zeta_x, \\ \varphi^- = \varphi\,(c_y < 0) = a_0^-\,(y)\,\zeta_x. \end{cases} \tag{3}$$

The velocity of the gas u and the current of an impulse p_{xy} are connected with the functions a_0^+ and a_0^- by the relationships

$$U = uv_w^{-1} = \frac{1}{4}\left(a_0^+ + a_0^-\right);\quad p_{xy} = \frac{a_0^+ - a_0^-}{4\sqrt{\pi}}.$$

We multiply (1) by the functions ζ_x, $\zeta_x\zeta_y$ and, using approximation (3), we average the result in the space of the velocity **c**. Then for Maxwell molecules [10] we will have

$$\partial U/\partial\tau + \partial P/\partial Y = 0;\quad \partial P/\partial\tau + \partial U/\partial Y = -P. \tag{4}$$

Here

$$\tau = t/t_f,\quad Y = y\sqrt{2}/l_f,\quad l_f = 2\mu/\rho v_w,\quad t_f = l_f/v_w,\quad P = p_{xy}/(\sqrt{2}p_\infty).$$

Substituting (3) into boundary condition (2), we obtain

$$\sigma U\,(\tau, y=0) + (2-\sigma)\sqrt{\frac{\pi}{2}}\,P\,(\tau, y=0) = \sigma\bar{w}. \tag{5}$$

To solve the system of equations (4) we use a Laplace transform [11]:

$$\widehat{U}\,(Y, S) = \int_0^\infty e^{-S\tau}U\,(Y,\tau)\,d\tau;\quad \widehat{P} = \int_0^\infty e^{-S\tau}P\,d\tau. \tag{6}$$

Applying the transform (6) to Eq. (4) and taking account of the initial and boundary conditions, we obtain

$$S\widehat{U} + \partial\widehat{P}/\partial Y = 0;\quad S\widehat{P} + \partial\widehat{U}/\partial Y = -\widehat{P}, \tag{7}$$

$$\widehat{P}(Y\to\infty) = \widehat{U}(Y\to\infty) = 0, \tag{8}$$

$$\sigma\widehat{U}\,(Y=0) + (2-\sigma)\sqrt{\frac{\pi}{2}}\,\widehat{P}\,(Y=0) = \sigma\,\bar{w}S^{-1}. \tag{9}$$

The solution of the system of equations (7) with the boundary conditions (8), (9) has the form

$$\widehat{U}\,(Y, S) = \frac{\sigma\bar{w}}{S}\,\frac{\sqrt{S+1}\exp(-\alpha Y)}{\sigma\sqrt{S+1} + (2-\sigma)\sqrt{\frac{\pi}{2}}\sqrt{S}};$$

$$\widehat{P}\,(Y, S) = \frac{\sigma\bar{w}}{S}\,\frac{\sqrt{S}\exp(-\alpha Y)}{\sigma\sqrt{S+1} + (2-\sigma)\sqrt{\frac{\pi}{2}}\sqrt{S}},\quad \alpha = \sqrt{S\,(S+1)}.$$

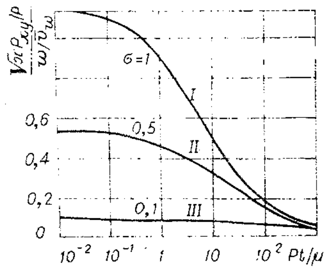

Fig. 3

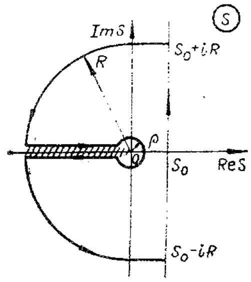

Fig. 4

To determine the inverse transforms of the functions $\hat{U}$ and $\hat{P}$, i.e., the inversions of the Laplace transform, we use an integration contour in the complex plane, shown in Fig. 1.

After simple transformations we obtain

$$U=\bar{w}\theta\left[1-\sqrt{\frac{2}{\pi}}\sigma(2-\sigma)\int_0^1\frac{\sqrt{1-z^2}\,e^{-\tau z^2}\cos\left(Yz\sqrt{1-z^2}\right)dz}{\sigma^2+z^2\varepsilon^2}-\frac{2\sigma^2}{\pi}\int_0^1\frac{\sqrt{1-z^2}\,e^{-\tau z^2}\sin\left(Yz\sqrt{1-z^2}\right)dz}{z(\sigma^2+z^2\varepsilon^2)}\right]; \tag{10}$$

$$P=\bar{w}\theta\left[\frac{2}{\pi}\sigma^2\int_0^1\frac{\sqrt{1-z^2}\,e^{-\tau z^2}\cos\left(Yz\sqrt{1-z^2}\right)dz}{\sigma^2+z^2\varepsilon^2}-\sqrt{\frac{2}{\pi}}\sigma(2-\sigma)\int_0^1\frac{ze^{-\tau z^2}\sin\left(Yz\sqrt{1-z^2}\right)dz}{\sigma^2+z^2\varepsilon^2}\right], \tag{11}$$

where θ is a Heaviside function [11],

$$\theta=\theta(\tau-Y),\ \ \theta(x>0)=1,\ \ \theta(x<0)=0;\ \ \varepsilon^2=\frac{\pi}{2}(2-\sigma)^2-\sigma^2.$$

With $y=0$, $\sigma=1$, formulas (10), (11) coincide with a solution obtained earlier for the case of a diffusionally reflecting wall [5].

Let us investigate in more detail the dependence $U(\tau, Y=0)$, $P(\tau, Y=0)$. For an interval of time much less than $t_f(\tau \ll 1)$, expanding the expressions under the integration sign in (10), (11) in series in terms of τ, and taking account of the tabulated values of the integrals [12], we will have

$$U=\bar{w}\theta(\tau)\left[\frac{\sigma}{(2-\sigma)\sqrt{\frac{\pi}{2}}+\sigma}-\frac{\sigma(2-\sigma)\sqrt{\pi/2}\,\tau}{2\left[(2-\sigma)\sqrt{\pi/2}+\sigma\right]}+0(\tau^2)\right];$$

$$P=\bar{w}\theta(\tau)\{(\sigma/(2-\sigma)\sqrt{\pi/2})-(\sigma^2\tau/2[(2-\sigma)\sqrt{\pi/2}+\sigma]^2)+0(\tau^2)\}.$$

A comparison with the exact solution of Eq. (1) near the limit $\tau\to 0$ [9] $U(\tau, Y=0)=(\sigma/2)+[\sigma(2-\sigma)/8]\tau+0(\tau^2)$; $P=1+0(\tau)$, shows that the difference in the value of the velocity $U(Y=0, \tau)$ does not exceed 5%, and, in the value of P, 10%. This exactness can be considerably increased, as is shown in [13] (with $\sigma=1$, $\tau \ll 1$), even after the first iteration of the zero approximation (3).

For intervals of time much larger than $t_f(\tau \gg 1)$, after an appropriate simplification of formulas (10), (11), we obtain

$$U(\tau, Y=0)=w\theta(\tau)\left[1-\frac{2-\sigma}{\sigma\sqrt{2\tau}}+0\left(\frac{1}{\tau}\right)\right];$$

$$P(\tau, Y=0)=\bar{w}\theta(\tau)\left[\frac{1}{\sqrt{2\tau}}+0\left(\frac{1}{\tau}\right)\right].$$

The time required for establishment of steady-state slip T_σ in a plane with incomplete accomodation $(\sigma<1)$ is σ^{-2} times greater than the time $T_1(\sigma=1)$,

$$T_\sigma/T_1=\sigma^{-2}.$$

To determine the values of the functions $U(\tau, Y=0, \sigma)$, $P(\tau, Y=0, \sigma)$ in the intermediate range $(\tau\sim 1)$ numerical integration of dependences (10), (11) is needed.

The results of calculations (for different values of σ) are given in Fig. 2 and Fig. 3.

In the region $\sigma \ll 1$, going over in formula (10) to the variable $\xi=z\sigma^{-1}$, we obtain

$$U(\tau, Y=0, \sigma)=\bar{w}\theta(\tau)\left[1-\frac{2\sqrt{2}}{\sqrt{\pi}}\int_0^\infty\frac{\exp(-\sigma^2\tau\xi)}{1+2\pi\xi^2}d\xi+0(\sigma)\right].$$

Analogy with the Propagation of Free Vibrations in an Electrical Line of Infinite Length

The differential equations describing the propagation of free vibrations in an electrical line have the form [14]

$$L\partial I/\partial t + \partial V/\partial y = -RI; \quad C\partial V/\partial t + \partial I/\partial y = GV, \tag{12}$$

where L is the inductance; C is the capacitance; G is the leakage; R is the ohmic resistance (L, C, G, R are referred to unit length); I is the strength of the current; V is the voltage.

It can be seen that the systems of equations (4) and (12) coincide if we set L = 1, C = 1, R = 1, C = 0, V = U, I = P.

The initial conditions U = P = 0 correspond to an open line at the moment of time $t \leq 0$, and the boundary conditions (5) and (8), to conditions at the ends of the line. For example, condition (8) corresponds to the absence of a reflected wave from the end of the line, while the condition (5) $V(t, y=0) = \varepsilon - z_0 I(t, y=0)$ means that the emf of the battery $\varepsilon = \bar{w}$ is made up of the emf at the start of the line V(y = 0) and the voltage drop in the ohmic resistance $Z_0\left(Z_0 = \sqrt{\frac{\pi}{2}}\frac{2-\sigma}{\sigma}\right)$.

Equations (12) can be brought to a single telegraphic equation with respect to V or I [14],

$$\partial^2 V/\partial t^2 - \partial^2 V/\partial y^2 + \partial V/\partial t = 0. \tag{13}$$

Thus, the solution of the system of equations (4) is equivalent to solution of the telegraphic equation (14), describing the propagation of free vibrations in an electrical line of finite length.

Unsteady-State Slip near a Surface with Almost Mirror Reflection ($\sigma \ll 1$) of Molecules

In the hydrodynamic solution of the problem of slip (i.e., with $\tau \gg 1$) the value of the derivative $\partial P/\partial t$ is small; therefore, Eq. (4) and the boundary condition (6) assume the form

$$\partial U/\partial \tau + \partial P/\partial Y = 0; \quad \partial U/\partial Y = -P; \tag{14}$$

$$\sigma V + (2-\sigma)\sqrt{\pi/2}\;\partial U/\partial Y = \sigma\bar{w}. \tag{15}$$

Applying a Laplace transform to relationships (14), (15), analogously to what has gone before we obtain

$$\hat{U} = \sigma\bar{w}\exp(-\sqrt{S}Y)/S[\sigma + (2-\sigma)\sqrt{\pi/2}\sqrt{S}].$$

The inverse transform $\hat{U}$ can be found from tables [15]

$$U = \bar{w}\left\{\operatorname{erfc}\left(\frac{Y}{\sqrt{2\tau}}\right) - \exp\left(\sqrt{\frac{2}{\pi}}\frac{\sigma Y}{2-\sigma}\right)\exp\left[\frac{2}{\pi}\frac{\sigma^2\tau}{(2-\sigma)^2}\right]\operatorname{erfc}\left(\sqrt{\frac{2}{\pi}}\frac{\sigma\sqrt{\tau}}{2-\sigma} + Y/2\sqrt{\tau}\right)\right\} \tag{16}$$

or by direct integration in the complex plane, along the contour illustrated in Fig. 4,

$$U = 1 - \frac{2\sigma\delta}{\pi}\int_0^\infty \frac{e^{-\tau z^2}\cos(Yz)\,dz}{\sigma^2 + z^2\delta^2} - \frac{2\sigma^2}{\pi}\int_0^\infty \frac{e^{-\tau z^2}\sin(Yz)\,dz}{z(\sigma^2 + z^2\delta^2)}. \tag{17}$$

Here $\delta^2 = \varepsilon^2 + \sigma^2$.

With $\sigma \to 0$, formulas (10) and (17) coincide. Thus, in the region of small values of σ ($\sigma \ll 1$) the solution of Eqs. (4) tends toward the hydrodynamic solution. This fact is not the result of chance. We shall show that at almost the whole mirror surface ($\sigma \ll 1$) the slip becomes appreciably different from zero only under hydrodynamic slip conditions ($\tau \gg 1$).

We multiply Eq. (1) by the function $\varphi(t, c, y)$ and integrate over all the variables. Taking account of the initial and boundary conditions for φ, we obtain

$$\frac{1}{2}\int_0^\infty \langle\tilde{\varphi}^2\rangle\,dy - \int_0^\infty dy\int_0^T \langle\tilde{\varphi}\mathscr{L}[\tilde{\varphi}]\rangle\,dt = \frac{\sigma}{2}\int_0^T\int_{c_y>0} \zeta_y(2\zeta_x - \varphi^-)[(2-\sigma)\varphi^- + 2\sigma\zeta_x]\,d\zeta dt, \tag{18}$$

where $\tilde{\varphi}=\varphi/w$, $\langle\varphi\rangle=\int e^{-\zeta^2}\varphi d\zeta$.

The left-hand part of the equality is a positively determined quantity, since $\langle\varphi\mathscr{L}[\varphi]\rangle<0$ [10]. Therefore, with $\sigma=0$, the functions $\varphi=0$. In addition, by virtue of (18) and of the inequality

$$\langle\varphi\mathscr{L}[\varphi]\rangle>\mu_1\langle\varphi^2\rangle\ ,\ \mu_1=\text{const}>0, \tag{19}$$

valid for "rigid" potentials [10], we obtain

$$\int_0^\infty\langle\tilde{\varphi}^2\rangle\,dy<\frac{\sigma t}{0.5+\mu_1}. \tag{20}$$

Thus, for the moments of time $\tau\sim 1$, slip (with an accuracy up to quantities on the order of σ) is absent, while, in the range $\tau\gg 1$, it corresponds to the hydrodynamic solution of the problem.

As a second example, let us consider the problem of unsteady-state thermal slip, formulated in [16]:

$$\frac{\partial\varphi}{\partial t}+c_y\frac{\partial\varphi}{\partial y}+\overline{T}_x c_x\left(\zeta^2-\frac{5}{2}\right)=\mathscr{L}[\varphi]; \tag{21}$$

$$\varphi(y,\ t=0)=\varphi_t\zeta_x\overline{T}_x;\ \varphi(y=0,\ t,\ c_y>0)=(1-\sigma)\varphi(c_y<0). \tag{22}$$

Here $\overline{T}_x$ is the gradient of the temperature in the direction x, "included" in the moment of time $t=0$, and, with $t>0$, maintained constant in value, both in the volume of the gas ($y>0$) and along the plane ($y=0$); $\overline{T}_x=T_w^{-1}(dT/dx)$; $\zeta_x\varphi_t\overline{T}_x$ is an Enske function [10].

It can be seen that, for the function $\psi=\varphi-\zeta_x\overline{T}_x\varphi_t$, Eq. (21) and conditions (22) coincide with the corresponding relationships of the Rayleigh problem, if, in the boundary condition (2), the "source" $2\sigma\zeta_x\overline{w}$ is replaced by the "sink" $\sigma\zeta_x\overline{T}_x\varphi_t$. Therefore, the preceding conclusions, obtained on the basis of relationships (18)-(20), are valid also for thermal slip with small values of the parameter σ. To determine the function U(t, y) in this case we must find the solution of the hydrodynamic equations (14) with the boundary condition with $y=0$, obtained, as in a number of articles, on the basis of solution of the steady-state variant of Eq. (1) in a Knudsen layer [16-18]:

$$U(t,y=0)=\eta_i l\frac{\partial U(t,y=0)}{\partial y}+\eta_T l\overline{T}_x,$$

where η_i, η_T are the coefficients of isothermal and thermal slip (of order 1); l is the length of the free-flight path.

Analogously to relationship (16) we obtain

$$U(\tau,Y)=\eta_T\overline{T}_x\left\{\operatorname{erfc}\left(\frac{Y}{2\sqrt{\tau}}\right)-\exp\left(\frac{Y}{\eta_i}+\frac{\tau}{\eta_i^2}\right)\operatorname{erfc}\left(\frac{\sqrt{\tau}}{\eta_i}+\frac{Y}{2\sqrt{\tau}}\right)\right\}.$$

From this, taking into account that, with $\sigma\to 0$, $\eta_T\approx\eta(\sigma=0)$ [16-18], $\eta_i\to(2/\sigma)\sqrt{\pi/2}$ [19], we will have

$$U=\eta_i\overline{T}_x\left[1-\exp\left(\frac{\tau\sigma^2}{2\pi}\right)\operatorname{erfc}\left(\frac{\sigma\sqrt{\tau}}{\sqrt{2\pi}}\right)\right]. \tag{23}$$

If the moment of time τ is given and $\sigma\to 0$, then, by virtue of (23), $U\sim\sigma\sqrt{\tau}\to 0$.

If the value of σ is given and $\tau\to 0$, then $U\sim 1-\frac{\sqrt{2}}{\sigma\sqrt{\tau}}\to 1$.

Thus, steady-state thermal slip at a mirror wall ($\sigma\to 0$) [16-18] should be understood as the limiting flow attainable with $\tau\to\infty$, $\sigma\to 0$ [16].

The author thanks V. V. Struminskii, V. N. Zhigulev, and V. P. Shidlovskii for their evaluation of the work.

LITERATURE CITED

1. G. G. Stokes, "On the effect of internal friction of fluids on the motion of pendulums," Trans. Cambridge Phil. Soc., 9, No. 8 (1851). Mathematical and Physics Papers, Cambridge (1901), pp. 1-141.
2. Rayleigh, "On the motion of solid bodies through viscous liquids," Phil. Mag., 21, 697-711 (1911).

3. H. T. Yang and L. Lees, Rayleigh's problem at low Mach number according to kinetic theory of gases," J. Math. Phys., 35, No. 3, 195-235 (1956).
4. E. P. Gross and E. A. Jackson, "Kinetic theory of the impulsive motion of an infinite plane," Phys. Fluids, 1, No. 4, 318-328 (1958).
5. Yu. A. Koshmarov, "Flow of a rarefied gas around a wall suddenly set into motion," Inzh. Zh., 3, No. 3, 433-441 (1963); V. P. Shidlovskii, Introduction to the Theory of a Rarefied Gas [in Russian], Izd. Nauka, Moscow (1965).
6. H. T. Yang and L. Lees, "The Rayleigh problem for low Reynolds numbers according to the kinetic theory of gases," in: Gasdynamics of Rarefied Gases [Russian translation], Izd. Inostr. Lit., Moscow (1963), pp. 325-375.
7. L. Trilling, "Asymptotic solution of the Boltzmann–Krook equation for the Rayleigh shear flow problem," Phys. Fluids, 7, No. 10, 1681-1691 (1964).
8. C. Cercignani and F. Sernagiotto, "Rayleigh's problem at low Mach numbers according to kinetic theory," in: Rarefied Gas Dynamics, Vol. 1, Academic Press, New York (1965), pp. 332-353.
9. M. Epstein, "Linearized Rayleigh's problem with incomplete surface accomodation," in: Rarefied Gas Dynamics, Academic Press, Vol. 1, New York (1969), pp. 255-265.
10. C. Cercignani, Mathematical Methods in Kinetic Theory, Plenum (1969).
11. A. V. Lykov, The Theory of Thermal Conductivity [in Russian], Izd. Vysshaya Shkola, Moscow (1967).
12. M. S. Gradshtein and I. M. Ryzhik, Tables of Integrals, Series, and Products, Academic Press (1966).
13. E. P. Gross and E. A. Jackson, "Kinetic models and the linearized Boltzmann equation," in: Mekhanika [Periodic collection of translations of foreign articles], No. 5 (1960), pp. 65-81.
14. M. A. Lavrent'ev and B. V. Shabat, Methods in the Theory of Functions of a Complex Variable [in Russian], Izd. Nauka, Moscow (1973).
15. V. A. Ditkin and P. I. Kuznetsov, Handbook on Operational Computation [in Russian], Izd. GITTL, Moscow–Leningrad (1951).
16. S. K. Loyalka and J. W. Cipolla, "Thermal creep slip with arbitrary accomodation at the surface," Phys. Fluids, 14, No. 8, 1656-1661 (1971).
17. Yu. I. Yalamov, I. N. Ivchenko, and B. V. Deryagin, Gasdynamic calculation of the rate of thermal slip of a gas near a solid surface," Dokl. Akad. Nauk SSSR, 177, No. 1, 74-76 (1967).
18. Yu. Yu. Abramov and G. G. Gladush, "Flow of a rarefied gas near an inhomogeneous heated surface," Izv. Akad. Nauk SSSR, Mekh. Zhidk. Gaza, No. 2, 20-29 (1970).
19. S. K. Loyalka, Approximate method in the kinetic theory," Phys. Fluids, 14, No. 11, 2291-2294 (1971).

MOTION OF AN ELECTROLYTE–GAS INTERFACE IN AN ELECTRICAL FIELD

V. P. Blinov and Yu. V. Zhilin

UDC 537.84

The article discusses the phenomenon of the motion of an electrolyte–gas interface, first observed on electrical levels. It gives a comparative characterization of the phenomena, for purposes of practical use, with a solution of problems of control by the position of a liquid–liquid interface.

While the mechanics of a liquid has been rather well studied at the present time, the problem of effective control by the position of a liquid has not yet been solved. It is a question here not of mechanical

Miass. Translated from Zhurnal Prikladnoi Mekhaniki i Tekhnicheskoi Fiziki, No. 6, pp. 25-28, November-December, 1975. Original article submitted July 16, 1974.